

Computing and Using the LU Decomposition

1 COMPUTING LU DECOMPOSITIONS.

Compute the LU decomposition of the following matrices.

$$(a) \begin{bmatrix} 2 & -1 \\ 4 & 0 \end{bmatrix}$$

$$(b) \begin{bmatrix} 2 & -1 \\ 4 & -2 \end{bmatrix}$$

$$(c) \begin{bmatrix} 2 & -1 & 3 \\ 4 & 0 & 7 \end{bmatrix}$$

$$(d) \begin{bmatrix} 2 & -1 & 3 \\ 4 & -2 & 7 \end{bmatrix}$$

$$(e) \begin{bmatrix} 2 & -1 & 3 \\ 4 & -4 & 7 \\ 0 & -2 & 4 \end{bmatrix}$$

$$(f) \begin{bmatrix} 2 & -1 \\ 4 & -4 \\ 0 & -2 \end{bmatrix}$$

$$(g) \begin{bmatrix} 2 & -1 & 3 \\ 4 & -4 & 7 \\ -2 & -1 & -2 \end{bmatrix}$$

$$(h) \begin{bmatrix} 2 & -1 & 3 \\ 4 & -2 & 4 \\ 4 & -2 & 8 \end{bmatrix}$$

2 USING THE LU DECOMPOSITION

Solve using the LU decompositions given below.

$$(a) \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 3 \\ 0 & 3 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -7 \\ -6 \end{bmatrix}$$

$$(b) \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} -2 & -3 & 1 \\ 0 & -2 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$(c) \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 0 & -2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 8 \\ 6 \end{bmatrix}$$

$$(d) \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 3 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

$$(e) \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \\ -5 \end{bmatrix}$$

$$(f) \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & -2 & 2 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 5 \end{bmatrix}$$

$$(g) \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ -5 \end{bmatrix}$$

$$(h) \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & -2 & 2 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

3 PUTTING IT ALL TOGETHER.

Compute the LU decomposition and use it to solve the following equations.

$$(a) \begin{bmatrix} 2 & 2 & -1 \\ -4 & -5 & 4 \\ 4 & 1 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 14 \end{bmatrix}$$

$$(b) \begin{bmatrix} 2 & 2 & -1 \\ -4 & -4 & 4 \\ 4 & 4 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 8 \end{bmatrix}$$

$$(c) \begin{bmatrix} 2 & 2 & -1 \\ -4 & -5 & 4 \\ 4 & 1 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 11 \end{bmatrix}$$

4 MATLAB

- Once you have entered a matrix in MatLab, you can access specific positions of the matrix using (row, col) notation. For example, the commands

```
1 >> A = [1 2 3; 4 5 6; 7 8 9]
2 >> A(2,3)
```

will give the output 6, which is the number in row 2 and column 3 of A.

You can also modify specific locations of a matrix this way. For example

```
3 >> A(2,3) = 20
```

will change the element in row 2, column 3 to be 20. Now the matrix A is $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 20 \\ 7 & 8 & 9 \end{bmatrix}$.

To access multiple elements of a matrix use start:end sequence notation. For example,

```
4 >> A(2,2:3)
```

will give output 5 20, which are the elements in row 2, columns 2-3. To get all of the elements in a row or column, use the : by itself. For example,

```
5 >> A(2,:) 
```

will give the output 4 5 20, which is all of row 2.

Using this, you can perform row operations on a matrix manually. For example, to change row 2 of our matrix to be (row 2) - 4*(row 1) we use the command

```
6 >> A(2,:) = A(2,:) - 4*A(1,:) % (Row 2) = (Row 2) - 4 * (Row 1)
```

The green text after the % symbol above is ignored by MatLab. It is a “comment” for the code, intended to help humans to read what is happening. **Good programmers use comments.**

After this command the matrix A will be $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & 8 \\ 7 & 8 & 9 \end{bmatrix}$. To complete the row reduction, we would use the commands

```
7 >> A(3,:) = A(3,:) - 7*A(1,:) % (Row 3) = (Row 3) - 7 * (Row 1)
8 >> A(3,:) = A(3,:) - 2*A(2,:) % (Row 3) = (Row 3) - 2 * (Row 2)
```

The resulting matrix is U (in the LU decomposition) while the multipliers 4, 7, and 2 are the numbers in L .

- MatLab can also compute LU decompositions directly using the lu command. Unfortunately what MatLab does is slightly more complicated than what we do. For numerical reasons MatLab wants to always choose its pivot to be as large as possible, and it will rearrange the rows of A to make this happen. So for example

```
9 >> [L,U] = lu([1 2 3; 4 5 20; 7 8 9])
```

will actually give the matrix U for the permuted matrix $\begin{bmatrix} 7 & 8 & 9 \\ 1 & 2 & 3 \\ 4 & 5 & 20 \end{bmatrix}$ instead of for the given matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 20 \\ 7 & 8 & 9 \end{bmatrix}$. Be careful!

- You can also use MatLab to easily solve the normal equation. The ' command takes the transpose of a matrix or vector. So for example

```
10 >> [1 0 1; 1 -1 2]'
```

gives the transpose $\begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix}^T = \begin{bmatrix} 1 & 1 \\ 0 & -1 \\ 1 & 2 \end{bmatrix}$. If you have already entered the matrix A and vector b, then the normal equation has solution

```
11 >> (A' * A) \ (A' * b)
```